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NASA TECHNICAL  
MEMORANDUM

NASA TM X- 64790

A REACTIVE TORQUE CONTROL LAW FOR GYROSCOPICALLY  
CONTROLLED SPACE VEHICLES

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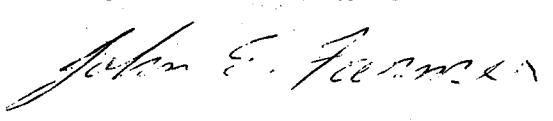
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August 24, 1973

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## TECHNICAL REPORT STANDARD TITLE PAGE

1. REPORT NO. NASA TM X- 64790	2. GOVERNMENT ACCESSION NO.	3. RECIPIENT'S CATALOG NO.	
4. TITLE AND SUBTITLE A Reactive Torque Control Law for Gyroscopically Controlled Space Vehicles		5. REPORT DATE August 24, 1973	
7. AUTHOR(S) John E. Farmer		6. PERFORMING ORGANIZATION CODE	
9. PERFORMING ORGANIZATION NAME AND ADDRESS George C. Marshall Space Flight Center Marshall Space Flight Center, Alabama 35812		8. PERFORMING ORGANIZATION REPORT #	
12. SPONSORING AGENCY NAME AND ADDRESS National Aeronautics and Space Administration Washington, D. C. 20546		10. WORK UNIT NO.	
		11. CONTRACT OR GRANT NO.	
		13. TYPE OF REPORT & PERIOD COVERED Technical Memorandum	
		14. SPONSORING AGENCY CODE	
15. SUPPLEMENTARY NOTES Prepared by Astrionics Laboratory, Science and Engineering			
16. ABSTRACT This report develops a method of control based on the reactive torques as seen by the individual CMG gimbals. The application of a torque to the gimbal of a CMG rotates the momentum vector and applies a torque to the spacecraft according to well-known laws. The response (rotation) of the vehicle produces a reverse or reaction torque opposing the torque producing the gimbal movement. The reactive torque and the pseudoinverse control schemes are contrasted in order to point out the simplicity of the first method. Simulation was performed only to the extent necessary to prove that reactive torque stabilization and control is feasible.			
17. KEY WORDS Reactive Torque CMG's Control Law Torque Requirements Rate Commanded Servos Gimbal Rates		18. DISTRIBUTION STATEMENT Unlimited - Unclassified 	
19. SECURITY CLASSIF. (of this report) Unclassified	20. SECURITY CLASSIF. (of this page) Unclassified	21. NO. OF PAGES 24	22. PRICE NTIS

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## DEFINITION OF SYMBOLS

<u>Symbol</u>	<u>Definition</u>
SG	Single gimbal
CMG	Control moment gyroscope
$H_n$	Momentum of the nth CMG
x, y, z	Orthogonal body fixed reference axes
i, j, k	Unit vectors along the x, y, z axes respectively
$\delta_n$	Gimbal angle of the nth CMG
$\dot{\delta}_n$	Angular velocity about command axis of the nth CMG
$\beta$	Base angle of pyramidal configuration
T	Torque
$\omega$	Angular rate
s	Sine
c	Cosine
$\dot{\delta}_c$	Commanded gimbal rates
S	Laplace operator
$\bar{}$	Superscript bar denotes a vector
$\tilde{}$	Superscript tilde denotes a vector in matrix notation
e	Subscript denotes error

## DEFINITION OF SYMBOLS (Concluded)

<u>Symbol</u>	<u>Definition</u>
g	Subscript denotes a command to the spacecraft
$\theta$	Difference between commanded and actual vehicle attitude
$\times$	Vector cross product
.	Dot — When not used as a superscript, denotes the vector dot product

# A REACTIVE TORQUE CONTROL LAW FOR GYROSCOPICALLY CONTROLLED SPACE VEHICLES

## SECTION I. INTRODUCTION

The reactive torque and the well-known pseudo or generalized inverse methods of control represent only two of many solutions [1] to the same problem. A comparison of the two methods is helpful in introducing the reactive torque concept, since the two control schemes are developed by approaching the problem from exactly opposite viewpoints.

The pseudoinverse is a result of the more direct approach to deriving a control law for gyroscopically torqued vehicles. A simplified block diagram of the generalized inverse scheme is shown in Figure 1. The control variables, i.e., the gimbal rate commands to the CMG's, are determined by the required torques about the body-fixed axes and then making use of the fact that [2]

$$\tilde{T} = - \sum_{n=1}^m (\dot{\delta}_n \times \bar{H}_n) = (B) \begin{pmatrix} \dot{\delta}_1 \\ \dot{\delta}_2 \\ \vdots \\ \dot{\delta}_m \end{pmatrix} = B \tilde{\delta} \quad , \quad (1)$$

where

$$n = 1, 2, \dots, m$$

$\tilde{T}$  = torque applied to each of the body-fixed axes by the combined action of the CMG's

$\dot{\delta}_n$  = angular gimbal rate of nth CMG

$\bar{H}_n$  = momentum of the CMG rotor

$B$  = matrix expressing the vector cross product and resolution to the vehicle body axes ( $B$  is a function of the momenta and gimbal angles).

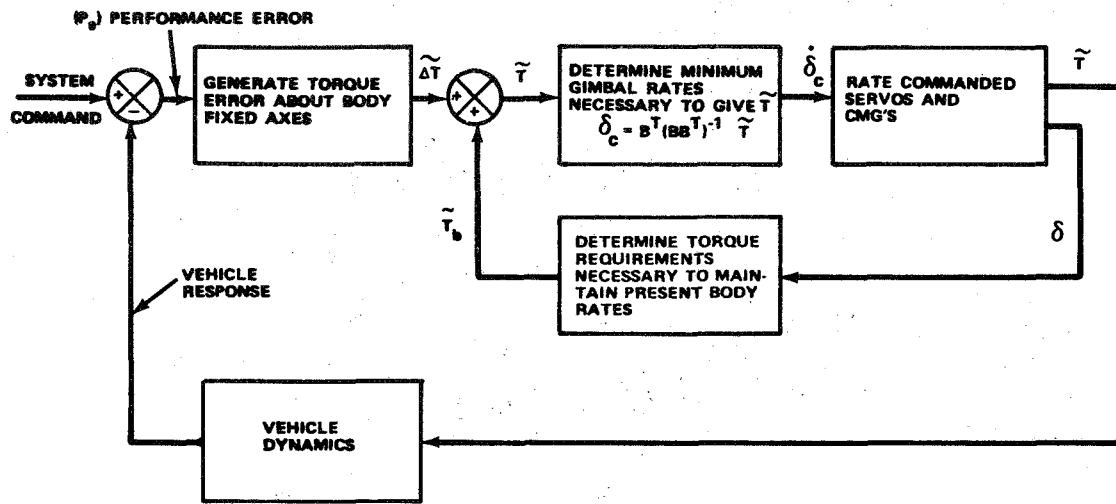


Figure 1. Simplified block diagram of pseudoinverse control scheme.

Assuming the commanded ( $\dot{\delta}_c$ ) and actual ( $\dot{\delta}$ ) gimbal rates are equal and solving equation (1) for the gimbal rates gives

$$\dot{\delta}_c = B^{-1} \tilde{T} \quad . \quad (2)$$

If there are four or more gyroscopes on board the vehicle, then  $B$  is a non-square matrix and many solutions for the  $\dot{\delta}_c$  matrix exist. A well-known solution to this problem is the pseudoinverse given by

$$\dot{\delta}_c = B^T (B B^T)^{-1} \tilde{T} \quad . \quad (3)$$

This solution is usually formulated to satisfy the side condition that the magnitude squared of the solution vector is a minimum.

The generalized inverse is a relatively complex method of control which requires considerable computing capability to calculate the inverse and to generate the torque requirements. Most of these complexities are circumvented by considering the control problem from the standpoint of the reactive torque produced on the individual CMG's by the rotating vehicle. If the vehicle has body rates ( $\bar{\omega}$ ) about the body-fixed axes, then the reactive torques on the CMG gimbals and opposing the torque output of the CMG servos are expressed by

$$\tilde{T}_R = \bar{\omega} \times \bar{H}_n = M\bar{\omega} \quad (4)$$

where  $M$  is a matrix expressing the vector cross product and the resolution onto the gimbal axis of each CMG. Figure 2 shows the block diagram of the reactive torque control scheme. It is a classic closed-loop control system operating on the error-nulling principle requiring that the outer loop be closed for stability.

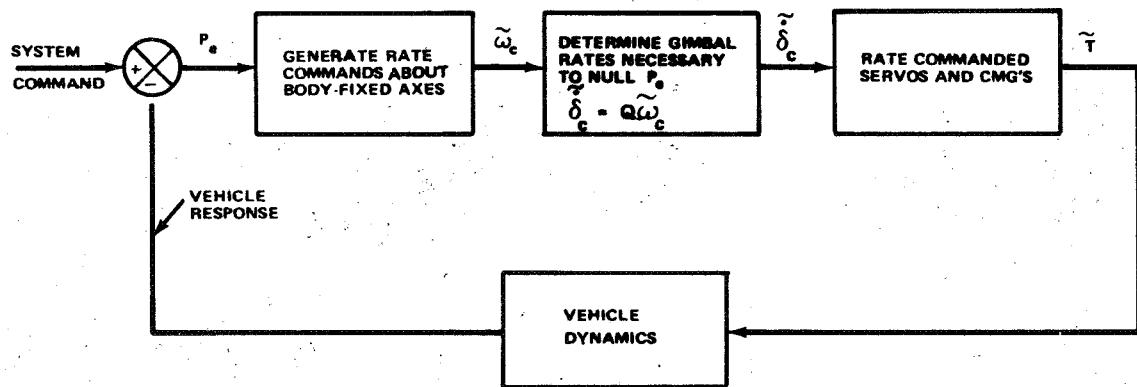


Figure 2. Simplified block diagram of reactive torque control scheme.

Unlike equation (2) the reactive torques are rigorously determined by equation (4), the option of satisfying some side condition does not exist, there is no need to generate a matrix inverse, and the rate commands ( $\bar{\omega}_c$ ) are easily calculated.

The reactive torque control scheme calculates the future system state as if the desired performance had already been achieved with the existing

gimbal angles. The CMG commands are then calculated on an individual basis and in a manner based upon the difference of the present and future states.

## SECTION II. COORDINATE SYSTEMS AND CMG ORIENTATION

Consider a vehicle controlled by four single-gimbal control moment gyroscopes (SGCMG) whose momentum vectors are known with respect to the body-fixed axes. As a baseline configuration\*, the SGCMG's are each mounted in the face of a pyramid with base angle  $\beta$  as shown in Figure 3. If the gimbal angles are defined as in Figure 3, then the positive gimbal angle rate vectors are in the direction shown and are perpendicular to the pyramid faces. The rate vectors are measured with respect to the body axes and the CMG momentum vectors as shown in the figure represent the null position, i.e., zero gimbal angle, for the CMG's.

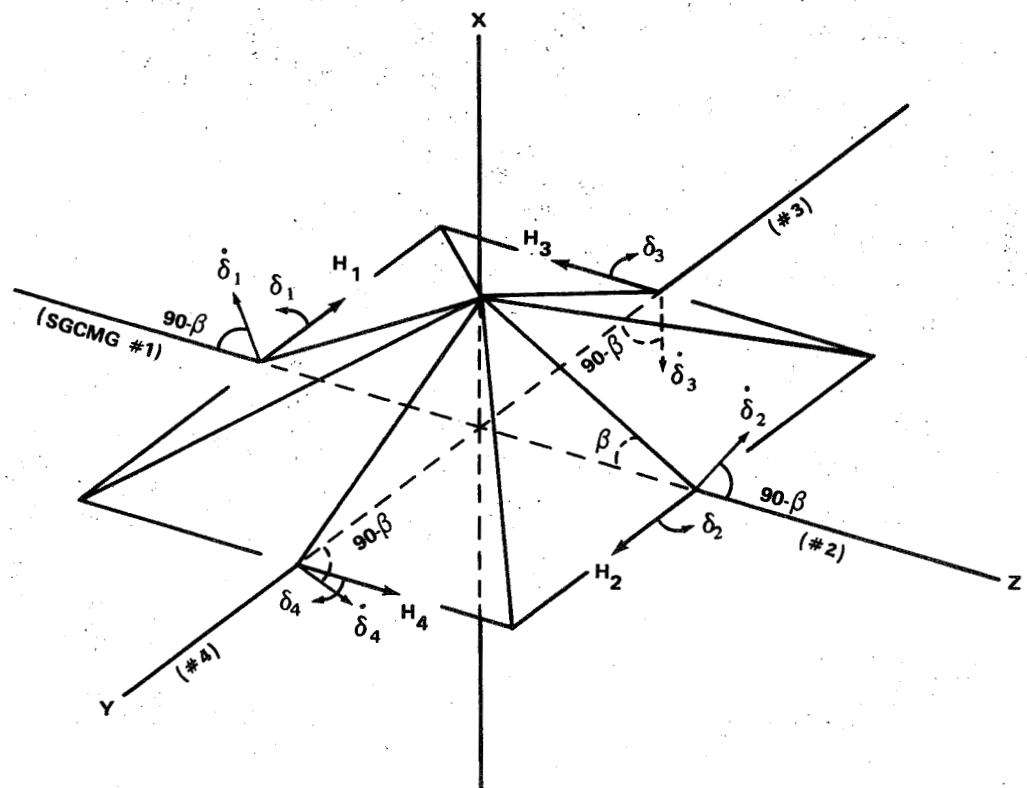


Figure 3. CMG Configuration.

\*TRW: TRW Technical Proposal Documents (Attitude Control and Determination System). May 1972.

Let  $\bar{\rho}_1$ ,  $\bar{\rho}_2$ ,  $\bar{\rho}_3$ , and  $\bar{\rho}_4$  be unit vectors which lie along the respective gimbal rate vectors such that

$$\begin{aligned}\bar{\rho}_1 &= c\beta \bar{i} - s\beta \bar{k} \\ \bar{\rho}_2 &= c\beta \bar{i} + s\beta \bar{k} \\ \bar{\rho}_3 &= -c\beta \bar{i} + s\beta \bar{j} \\ \bar{\rho}_4 &= -c\beta \bar{i} - s\beta \bar{j}\end{aligned}\tag{5}$$

where  $\bar{i}$ ,  $\bar{j}$ , and  $\bar{k}$  are unit vectors along the x, y, and z axes, respectively.

If the vehicle body rates are known (from onboard sensors) and are defined to be

$$\tilde{\omega}_v = \begin{pmatrix} \omega_x \bar{i} \\ \omega_y \bar{j} \\ \omega_z \bar{k} \end{pmatrix} ; \tag{6}$$

then the vector cross product indicated in equation (4) becomes

$$\bar{T}_R = \bar{\omega}_v \times \bar{H}_n \tag{7}$$

For future purposes, the vector multiplication as shown in equation (6) will not be used. The subscripts x, y, and z will be understood to denote scalars in the  $\bar{i}$ ,  $\bar{j}$ , and  $\bar{k}$  directions. Equation (6) will be written as

$$\tilde{\omega}_v = \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} . \tag{8}$$

### SECTION III. DERIVATION OF EQUATIONS AND TRANSFORMATIONS

The reactive torques have been defined as those components of  $\bar{\omega}_v \times \bar{H}_n$  which are parallel to the individual gimbal axes defined by the  $\bar{\rho}_n$  vectors.

The reactive torque vector matrix, equation (7), is most easily computed by forming the vector cross products shown in Table 1.

TABLE 1. REACTIVE TORQUES BEFORE RESOLUTION  
TO CMG GIMBAL AXES

$\bar{x}$	$\bar{H}_{x1}$	$\bar{H}_{x2}$	$\bar{H}_{x3}$	$\bar{H}_{x4}$
$\bar{z}_x$	0	0	0	0
$\bar{z}_y$	$H_1 s \delta_1 s \beta \omega_y \bar{k}$	$H_2 s \delta_2 s \beta \omega_y \bar{k}$	$H_3 s \delta_3 s \beta \omega_y \bar{k}$	$H_4 s \delta_4 s \beta \omega_y \bar{k}$
$\bar{z}_z$	$-H_1 s \delta_1 s \beta \omega_z \bar{j}$	$-H_2 s \delta_2 s \beta \omega_z \bar{j}$	$-H_3 s \delta_3 s \beta \omega_z \bar{j}$	$-H_4 s \delta_4 s \beta \omega_z \bar{j}$

$\bar{x}$	$\bar{H}_{y1}$	$\bar{H}_{y2}$	$\bar{H}_{y3}$	$\bar{H}_{y4}$
$\bar{z}_x$	$-H_1 c \delta_1 \omega_x \bar{k}$	$H_2 c \delta_2 \omega_x \bar{k}$	$-H_3 s \delta_3 c \beta \omega_x \bar{k}$	$H_4 s \delta_4 c \beta \omega_x \bar{k}$
$\bar{z}_y$	0	0	0	0
$\bar{z}_z$	$H_1 c \delta_1 \omega_z \bar{i}$	$-H_2 c \delta_2 \omega_z \bar{i}$	$H_3 s \delta_3 c \beta \omega_z \bar{i}$	$-H_4 s \delta_4 c \beta \omega_z \bar{i}$

$\bar{x}$	$\bar{H}_{z1}$	$\bar{H}_{z2}$	$\bar{H}_{z3}$	$\bar{H}_{z4}$
$\bar{z}_x$	$H_1 s \delta_1 c \beta \omega_x \bar{j}$	$-H_2 s \delta_2 c \beta \omega_x \bar{j}$	$H_3 c \delta_3 \omega_x \bar{j}$	$-H_4 c \delta_4 \omega_x \bar{j}$
$\bar{z}_y$	$-H_1 s \delta_1 c \beta \omega_y \bar{i}$	$H_2 s \delta_2 c \beta \omega_y \bar{i}$	$-H_3 c \delta_3 \omega_y \bar{i}$	$H_4 c \delta_4 \omega_y \bar{i}$
$\bar{z}_z$	0	0	0	0

Note: The table is read in the following manner. The vector cross product of  $\bar{\omega}_y$  and  $\bar{H}_{z4}$  is  $\bar{\omega}_y \times \bar{H}_{z4} = H_4 c \delta_4 \omega_y \bar{i}$

The values of the table are calculated by writing the CMG momenta in vector notation as

$$\bar{H}_n = H_{xn} \bar{i} + H_{yn} \bar{j} + H_{zn} \bar{k} \quad (9)$$

and then forming the cross product of  $\bar{\omega}_v$  with each of the momentum components of each of the four SGCMG's. The values shown in the table represent all reactive torques before resolution to the CMG gimbal axes.

Using Table 1 and equation (5), the reactive torques on the individual SGCMG's produced by the rotating vehicle and opposing the torques driving the gimbals can be expressed by

$$\tilde{T}_R = \begin{pmatrix} \bar{T}_{R1} \\ \bar{T}_{R2} \\ \bar{T}_{R3} \\ \bar{T}_{R4} \end{pmatrix} = \begin{pmatrix} [(\bar{\omega}_v \times \bar{H}_1) \cdot \bar{\rho}_1] \bar{\rho}_1 \\ [(\bar{\omega}_v \times \bar{H}_2) \cdot \bar{\rho}_2] \bar{\rho}_2 \\ [(\bar{\omega}_v \times \bar{H}_3) \cdot \bar{\rho}_3] \bar{\rho}_3 \\ [(\bar{\omega}_v \times \bar{H}_4) \cdot \bar{\rho}_4] \bar{\rho}_4 \end{pmatrix} = (K) (\tilde{\omega}_v) \quad (10)$$

where K is the matrix defined by

$$(K) = \begin{pmatrix} H_1 c \delta_1 s \beta & -H_1 s \delta_1 & H_1 c \delta_1 c \beta \\ H_2 c \delta_2 s \beta & H_2 s \delta_2 & -H_2 c \delta_2 c \beta \\ H_3 c \delta_3 s \beta & H_3 c \delta_3 c \beta & -H_3 s \delta_3 \\ H_4 c \delta_4 s \beta & -H_4 c \delta_4 c \beta & H_4 s \delta_4 \end{pmatrix} \quad (11)$$

The reactive torque vectors as defined by equation (3) can best be visualized by examining the vector diagram of an individual SGCMG. Figure

4 represents the reactive torques and body rates as seen by SGCMG number 1 when  $0 < \delta_1 < \pi/2$  radians.

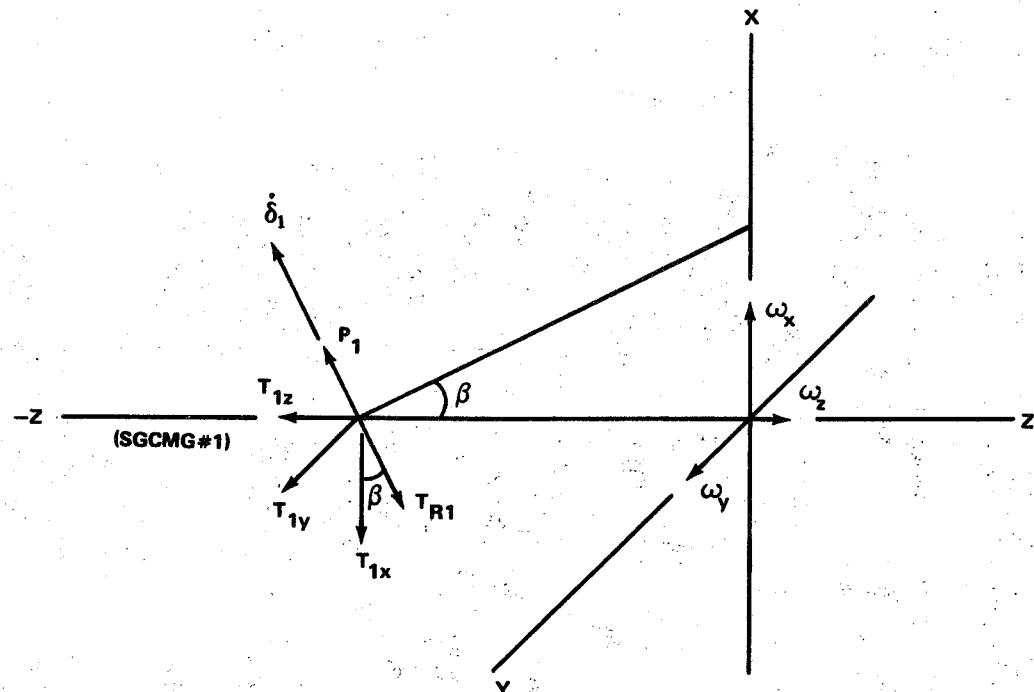


Figure 4. Reactive torques and body rates as seen by CMG number 1.

From Table 1

$$\bar{T}_{1x} = \bar{\omega}_z \times \bar{H}_{y1} + \bar{\omega}_y \times \bar{H}_{z1} = (H_1 c \delta_1 \omega_z - H_1 s \delta_1 c \beta \omega_y) \bar{i}$$

$$\bar{T}_{1y} = \bar{\omega}_z \times \bar{H}_{x1} + \bar{\omega}_x \times \bar{H}_{z1} = (H_1 s \delta_1 c \beta \omega_x - H_1 s \delta_1 s \beta \omega_x) \bar{j} \quad (12)$$

$$\bar{T}_{1z} = \bar{\omega}_y \times \bar{H}_{x1} + \bar{\omega}_x \times \bar{H}_{y1} = (H_1 s \delta_1 s \beta \omega_y - H_1 c \delta_1 \omega_x) \bar{k}$$

Since  $\bar{T}_{1y}$  is perpendicular to  $\bar{P}_1$ , it contributes nothing to the value of  $\bar{T}_{r1}$ .

From Figure 4 the magnitude of  $\bar{T}_{r1}$  is

$$T_{R1} = T_{1X} \cos \beta - T_{1Z} \sin \beta \quad (13)$$

which is identical to

$$T_{R1} = (\bar{\omega}_v \times \bar{H}_1) \cdot \bar{p}_1 \quad (14)$$

## SECTION IV. SYNTHESIS OF THE STEERING LAW

Assume the driving torques on the gimbals to be produced by rate-commanded servos whose outputs are the torques defined by

$$(\tilde{T}_m) = \begin{pmatrix} \bar{T}_{m1} \\ \bar{T}_{m2} \\ \bar{T}_{m3} \\ \bar{T}_{m4} \end{pmatrix} \quad (15)$$

Since the CMG's are single gimbal, these torques are fixed in direction relative to the body axes and lie along the gimbal axes defined by the unit  $\bar{p}$  vectors.

Let the gimbal inertias (which may or may not be identical) be defined as that inertia "seen" by the rate commanded servos. Symbolically the gimbal inertias will be defined by the equation

$$(I_g) = \begin{pmatrix} I_{g1} & 0 & 0 & 0 \\ 0 & I_{g2} & 0 & 0 \\ 0 & 0 & I_{g3} & 0 \\ 0 & 0 & 0 & I_{g4} \end{pmatrix} \quad (16)$$

If  $\dot{\delta}_n$  now represents the angular gimbal rates as shown in Figure 3, then

$$\tilde{(\dot{\delta})} = \begin{pmatrix} \dot{\delta}_1 \\ \dot{\delta}_2 \\ \dot{\delta}_3 \\ \dot{\delta}_4 \end{pmatrix} \quad (17)$$

and

$$S \tilde{(\dot{\delta})} = (I_g)^{-1} (\tilde{T}_m - \tilde{T}_R) \quad (18)$$

where  $S$  is the Laplace operator and, as used, signifies the differentiation of  $\tilde{\delta}$  with respect to time.

Equation (18) offers a convenient method of commanding the vehicle to rotate in some prescribed manner. It is reasonable to assume that the commanded and the actual gimbal rates are essentially equal. If the commanded gimbal rates are

$$\tilde{(\dot{\delta})}_c = \begin{pmatrix} \dot{\delta}_{1c} \\ \dot{\delta}_{2c} \\ \dot{\delta}_{3c} \\ \dot{\delta}_{4c} \end{pmatrix}, \quad (19)$$

then equation (18) becomes

$$\tilde{\dot{\delta}}_c = (S I_g)^{-1} (\tilde{T}_m - \tilde{T}_R) \quad . \quad (20)$$

Equation 20 defines the manner in which the torque requirements will be distributed among the various CMG's. If  $\tilde{T}_R$  is allowed to change by some amount  $\Delta \tilde{T}_R$  then the control law forces a corresponding change in the gimbal rate command according to equation (20).

Let

$$\tilde{T}_R = \tilde{T}_R + \Delta \tilde{T}_R \quad , \quad (21)$$

then substitution into equation (20) gives

$$\begin{aligned} \tilde{\dot{\delta}}_c + \Delta \tilde{\dot{\delta}}_c &= (S I_g)^{-1} (\tilde{T}_m - \tilde{T}_R - \Delta \tilde{T}_R) \\ &= (S I_g)^{-1} (\tilde{T}_m - \tilde{T}_R) - (S I_g)^{-1} (\Delta \tilde{T}_R) \end{aligned}$$

It now follows that

$$\Delta \tilde{\dot{\delta}}_c = - (S I_g)^{-1} (\Delta \tilde{T}_R) \quad . \quad (22)$$

Assume now that it is desired to command a change in the body rate vector  $\bar{\omega}_v$ . Let the commanded body rates be

$$(\tilde{\omega}_g) = \begin{pmatrix} \omega_{xg} \\ \omega_{yg} \\ \omega_{zg} \end{pmatrix} \quad . \quad (23)$$

For a given change in the vehicle body rates the corresponding change in the reactive torques can be calculated from equation (10) as follows

$$\Delta \tilde{T}_R = (K) (\tilde{\omega}_g) - (K) (\tilde{\omega}_v) = (K) (\tilde{\omega}_g - \tilde{\omega}_v) \quad . \quad (24)$$

Let the rate error matrix be

$$(\tilde{\omega}_e) = (\tilde{\omega}_v - \tilde{\omega}_g) \quad . \quad (25)$$

The substitution of equations (24) and (25) into (22) gives the result

$$\Delta \tilde{\dot{\delta}}_c = (S I_g)^{-1} (K) (\tilde{\omega}_e) \quad . \quad (26)$$

Equation (26) calculates the necessary adjustment in the gimbal rate commands to cause the body rate errors  $(\tilde{\omega}_e)$  to approach zero.

No provision has been made at this point to determine the orientation of the vehicle. For this reason, a more general formulation of equation (26) is

$$\Delta \tilde{\dot{\delta}}_c = (S I_g)^{-1} (K) (\tilde{\omega}_c) \quad . \quad (27)$$

If the initial values  $(\tilde{\dot{\delta}}_{ci})$  of the commanded gimbal rates are known then

$$\tilde{\dot{\delta}}_c = \tilde{\dot{\delta}}_{ci} + \Delta \tilde{\dot{\delta}}_c$$

The purpose of the introduction of the  $\tilde{\omega}_c$  matrix is to make the system dependent upon the attitude error as well as the attitude rate error defined by equation (25). Since small signal orientation errors of the vehicle can be

determined by integrating the body rates, the position error matrix ( $\tilde{\Theta}_e$ ) can also be defined in terms of equation (25)

$$(\tilde{\Theta}_e) = \frac{1}{S} (\tilde{\omega}_e) = \begin{pmatrix} \omega_x - \omega_{xg} \\ S \\ \omega_y - \omega_{yg} \\ S \\ \omega_z - \omega_{zg} \\ S \end{pmatrix} = \begin{pmatrix} \Theta_{ex} \\ \Theta_{ey} \\ \Theta_{ez} \end{pmatrix} . \quad (28)$$

If the attitude rate and position control gains are also included in the definition of  $\tilde{\omega}_c$  then

$$\tilde{\omega}_c = (a_1)(\tilde{\omega}_e) + (a_0)(\tilde{\Theta}_e) = \begin{pmatrix} \omega_{xc} \\ \omega_{yc} \\ \omega_{zc} \end{pmatrix} \quad (29)$$

where

$$(a_1) = \begin{pmatrix} a_{1x} & 0 & 0 \\ 0 & a_{1y} & 0 \\ 0 & 0 & a_{1z} \end{pmatrix} ; \quad (a_0) = \begin{pmatrix} a_{0x} & 0 & 0 \\ 0 & a_{0y} & 0 \\ 0 & 0 & a_{0z} \end{pmatrix} .$$

The control equation (27) used in conjunction with equation (29) produces the simplified system block diagram shown in Figure 5. The individual blocks have been divided into several major areas by the dashed lines for the purpose of a clearer definition or to indicate where the functions are performed in the spacecraft.

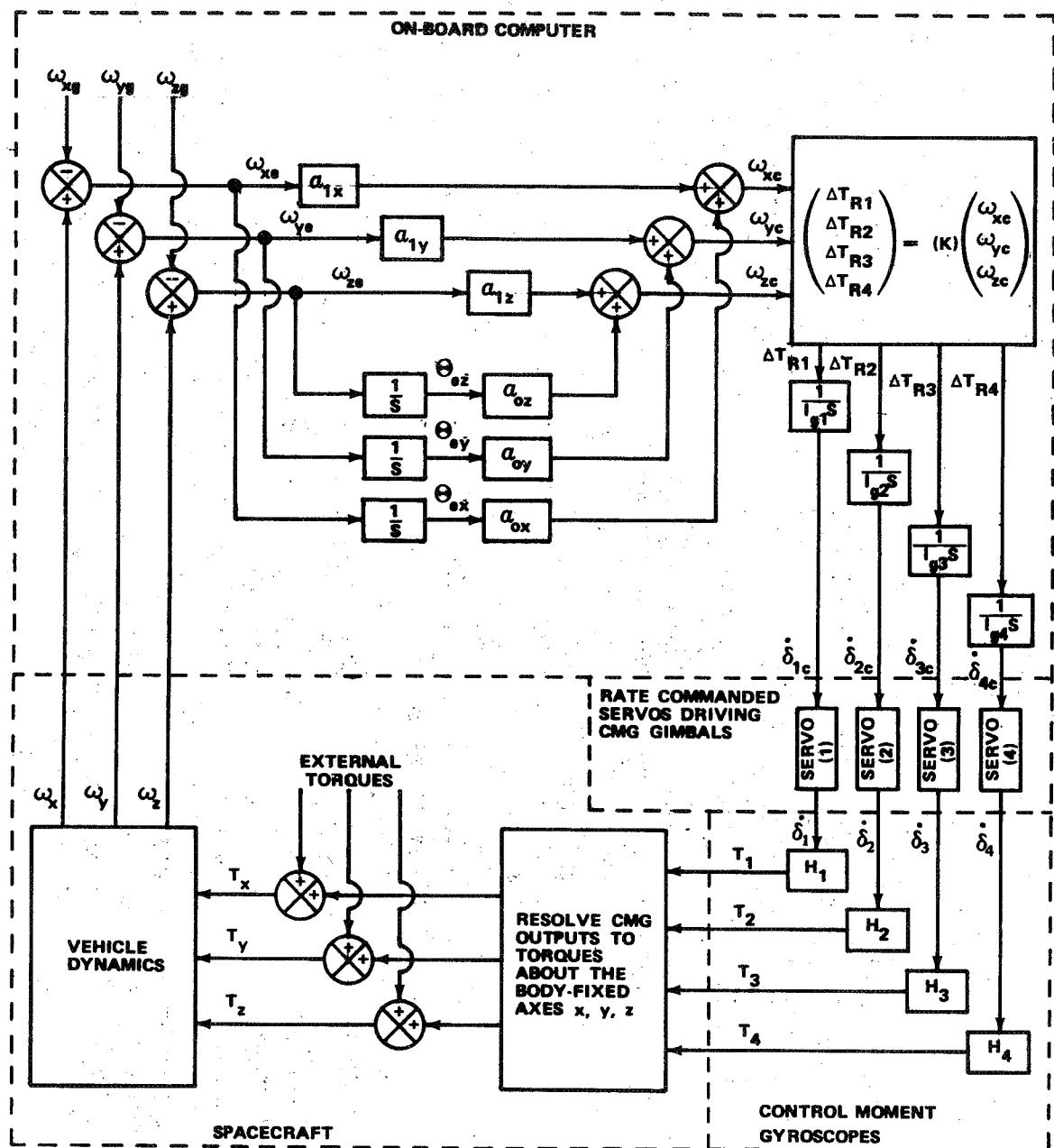


Figure 5. Simplified block diagram of reactive torque control scheme.

## SECTION V. COMMENTS AND CONCLUSIONS

Single-gimbal control moment gyroscopes were selected for this report in order to keep the equations as simple as possible and still illustrate the control principles involved. The reactive torque-control method will also apply to double gimbal CMG's and for any reasonable arrangement of the individual gyroscopes.

While it is doubtful that the gyroscopes are commanded in a way which minimizes the sum of the individual gimbal rates (as in the pseudoinverse method), the reactive torque method does command the gimbals in a gradient manner which gives the smallest commands to the least effective gyroscopes.

Normally, the vehicle body rates will be determined by rate gyroscopes. In order to null any errors which may develop, onboard updates of attitude rate and position may be accomplished by means of star trackers and sun sensors. The outputs of these sensors would be used to modify the rate gyro outputs and to determine the orientation of the vehicle.

Singularity avoidance has not been discussed in this report. Singularities do exist and should be the next point of investigation for anyone considering implementing this method of control.

The reactive torque control law has been simulated only to the point of showing that it is a feasible method of controlling a space vehicle. It represents a very simple system when compared to most existing methods and should be easily implemented.

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## APPROVAL

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By John E. Farmer

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

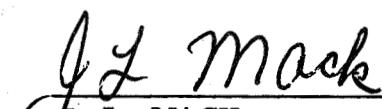
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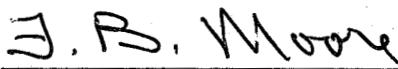
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